



Game Theory Intro

Game Theory Course: Jackson, Leyton-Brown & Shoham

Defining Games - Key Ingredients

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 - People? Governments? Companies? Somebody employed by a Company?...



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- Actions: what can the players do?
 - Enter a bid in an auction? Decide whether to end a strike? Decide when to sell a stock? Decide how to vote?
- Payoffs: what motivates players?
 - Do they care about some profit? Do they care about other players?...





Defining Games - Two Standard Representations

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 - But strategies encode many things...
- Extensive Form Includes timing of moves (later in course)
 - Players move sequentially, represented as a tree
 - Chess: white player moves, then black player can see white's move and react...
 - Keeps track of what each player knows when he or she makes each decision
 - Poker: bet sequentially what can a given player see when they bet?



• Finite, *n*-person normal form game: $\langle N, A, u \rangle$:



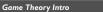
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- Utility function or Payoff function for player i: u_i : A → ℝ
 u = (u₁,..., u_n), is a profile of utility functions



Normal Form Games - The Standard Matrix Representation



- Writing a 2-player game as a matrix:
 - "row" player is player I, "column" player is player 2
 - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
 - cells listing utility or payoff values for each player: the row player first, then the column

Games in Matrix Form

Here's the TCP Backoff Game written as a matrix



$$C$$
 D

$$\begin{array}{c|c|c} C & -1, -1 & -4, 0 \\ \hline D & 0, -4 & -3, -3 \end{array}$$

A Large Collective Action Game



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- Action set for player $i A_i = \{Revolt, Not\}$
- Utility function for player *i*:

•
$$u_i(a) = 1$$
 if $\#\{j : a_j = Revolt\} \ge 2,000,000$

• $u_i(a) = -1$ if $\#\{j : a_j = Revolt\} < 2,000,000$ and $a_i = Revolt$

$$\bullet \ \ u_i(a) = 0 \quad \ \text{if} \ \#\{j: a_j = Revolt\} < 2,000,000 \ \text{and} \ a_i = Not$$

Summary: Defining Games

- For Now: Normal Form (Strategic Form, Matrix Representation...)
 - Players: N
 - Actions: A_i
 - Payoffs: u_i
- Later: Extensive Form
 - Timing: in what order do things happen?
 - Information: what do players know when they act

